

nts of the Fermi energy and of the  
of the  $d$  bands.

| Derived from<br>dHvA <sup>b</sup> | DFJ <sup>a</sup> |
|-----------------------------------|------------------|
| 0.5                               | -0.85            |
| 0.3                               | -0.86            |

$\Delta V/V$  is the relative change of the volume

$$\Delta V/V = \partial(\ln b_d)/\partial \epsilon_{yz}. \quad (20)$$

the list of strain coefficients  
ulate  $\partial L_1/\partial \epsilon_{yz}$ . Its numerical  
is 24% lower than the one  
periments.

ow the theoretical coefficient  
changing the assumption  
(a) gives the dependence of  
(0) to be valid. Figure 13(b)  
 $\partial(\ln H_{\phi d})/\partial \epsilon_{yz}$  using  $\partial b_d/\partial \epsilon_{yz}$   
atomic  $d$  functions.<sup>37</sup>  
on potentials  $\partial(E_F - L_3^u)/\partial \epsilon_{yz}$   
at 2.1 eV and  $\partial(L_1 - E_F)/\partial \epsilon_{yz}$   
) are used to calculate the  
and  $E_F$  relative to  $\Gamma_1$ . The  
y than the large experiment  
ation potential might sug  
er is due to the uncertainty  
e, we note that the relativ  
ental value from the true  
ual for the volume and the  
potentials.

amental volume deformation  
the sum  $\partial(L_1 - L_3^u)/\partial \epsilon_{yz}$  can  
 $\partial(\ln E_d)/\partial \epsilon_{yz}$ . In doing so, we  
ation factor  $\Omega^{-1/2}$  of  $b_d$  and

$$\Delta V/V = \partial(\ln b_d)/\partial k, \quad (21)$$

Eq. (20). There are several  
One possibility is to use the  
calculated with atomic  
 $(L_1 - L_3^u)/\partial \epsilon_{yz}$  which is 24%  
ental one. Another choice  
 $\partial(L_1 - L_3^u)/\partial \epsilon_{yz}$  until the theoretic  
the experimental one [Fig. 13(b)]  
 $\partial k$  by the same factor, and  
due of  $\partial(L_1 - L_3^u)/\partial \epsilon_{yz}$ . The  
turns out to be the same  
does not depend drastically  
 $b_d$  and  $H_{\phi d}$ . Its numerical  
In Fig. 14,  $\partial(L_1 - L_3^u)/\partial \epsilon_{yz}$   
the strain coefficients of

and  $H_{\phi d}$  in a fashion equivalent to that of Fig. 13. The  
dependence on  $\partial(\ln E_d)/\partial \epsilon_{yz}$  is also included [Fig. 14(b)].

In calculating the volume coefficient of  $E_F$ , we use  
the experimental value of  $\partial(E_F - L_3^u)/\partial \epsilon_{yz}$ , the volume  
coefficient of  $E_d$  as calculated above, and Eq. (7), to-  
gether with the strain coefficients of the tight binding  
parameters (Table V). The resulting value of  $\partial(\ln E_F)/\partial \epsilon_{yz}$   
is given in Table VII.

Assuming no strain dependence of  $b_d$  and  $H_{\phi d}$  at all  
except for  $\Omega^{-1/2}$ , we find the theoretical values of  
 $\partial L_1/\partial \epsilon_{yz}$  and  $\partial(L_1 - L_3^u)/\partial \epsilon_{yz}$  to be 62 and 50% of the  
corresponding experimental numbers [Eqs. 13(a) and  
14(a)], respectively. This part of the deformation  
potentials is mainly due to the strain dependence of  $k^2$   
and, for hydrostatic deformation, to the strain depen-  
dence of  $\Omega^{-1/2}$ .

### Discussion

The preceding analysis dealt with the observed  
structure in  $W_{ij}$ . A legitimate question is whether the  
energy bands predict more structure than actually  
observed. Pure shear strain will produce a significant  
change in  $\epsilon_2$  only for strongly or moderately localized  
transitions. Moreover, even if the transitions are  
localized but have  $k$  vectors of low symmetry (i.e.,  
neither parallel to [001] nor to [111]), there will be a  
signal for both trigonal and tetragonal strain (Table  
III) and the signal will tend to be small. Looking for  
localized  $\Delta$ ,  $X$ ,  $\Lambda$ , and  $L$  singularities only, we expect  
the  $X_5 \rightarrow X_4'$  and the  $FS \rightarrow L_1$  transitions to show up  
between 2 and 5.5 eV, as they do, i.e., the measure-  
ments are complete. On the other hand, hydrostatic  
strain will produce a signal for nonlocalized transitions  
too. Experimental examples are the maximum in  
 $W_{11} + 2W_{12}$  at 2.1 eV and the shoulder at 4.8 eV.

The energies of the identified transitions agree to  
within  $\pm 0.1$  eV with the corresponding difference of  
the eigenvalues, calculated with Chodorow's<sup>26</sup> potential.  
Band-structure calculations based on potentials differ-  
ent from that of Chodorow deviate from experiment  
by as much as 1.5 eV. Table VIII compares the energies  
of the experimentally observed transitions with pre-  
dictions of different calculations.<sup>13,14,42-44</sup> There are  
other experimental results which agree most closely  
with the result of the  $E(k)$  calculation based on  
Chodorow's potential, the most important of which is  
the area of the neck, measured with the de Haas-van  
Alphen effect. The experimental numbers which were  
re-examined recently<sup>45,46</sup> agree with the calcula-  
tion<sup>13,14,42</sup> to within 11%. For calculations with other

<sup>42</sup> J. S. Faulkner, H. L. Davis, and H. W. Joy, Phys. Rev. 161, 1656 (1967).

<sup>43</sup> H. L. Davis, J. S. Faulkner, and H. W. Joy, Phys. Rev. 167, 601 (1968).

<sup>44</sup> E. C. Snow and J. T. Waber, Phys. Rev. 157, 570 (1967).

<sup>45</sup> J. P. Jan and M. Templeton, Phys. Rev. 161, 556 (1967).

<sup>46</sup> W. J. O'Sullivan and J. W. Schriber, Cryogenics 7, 118 (1967).

TABLE VIII. Energies of observed transitions in eV.

| Energy        | Experiment     | Chodorow <sup>a</sup> $L$ -dependent <sup>b</sup> | Watson <sup>c</sup> | Self-consistent <sup>d</sup> |     |
|---------------|----------------|---|---------------------|------------------------------|-----|
| $E_F - L_3^u$ | 2.1 $\pm$ 0.1  | 2.1   | 2.3                 | 1.6                          | 3.2 |
| $X_4' - X_5$  | 4.0 $\pm$ 0.1  | 4.0   | 4.7                 | 3.1                          | 5.5 |
| $L_1 - E_F$   | 4.15 $\pm$ 0.1 | 4.0   | 5.15                | 3.9                          |     |

<sup>a</sup> References 13 and 14.

<sup>b</sup> References 13.

<sup>c</sup> References 42 and 43.

<sup>d</sup> Table II of Ref. 44.

potentials one might not get contact of the Fermi sur-  
face with the [111] face of the BZ at all.<sup>42</sup>

Thus, the experimental evidence for the superiority  
of the band structure calculated with Chodorow's  
potential is overwhelming. However, there is no  
theoretical formalism known today which tells us that  
we have to choose just this potential. For example, a  
self-consistent augmented-plane-wave calculation as  
the one reported by Snow and Waber<sup>44</sup> will agree with  
the experimental results once the exchange term is  
properly adjusted, but there is no theoretical justifica-  
tion for such an adjustment.

Zallen<sup>47</sup> measured the change of the reflectance with  
volume applying hydrostatic pressure directly to the  
crystal. His results are also listed in Table VI. He  
could quote only a lower limit for the deformation  
potential of the 2.1-edge. Our method is much more  
sensitive here because the large slope of the edge pro-  
duces a large  $\Delta \epsilon_2$  even for the small deformation poten-  
tial. The two experiments are of comparable accuracy  
in terms of energy shifts for the 4.3-eV edge. The  
modulation experiment lost part of its advantage here  
because the slope is smaller and the slope of the back-  
ground unknown. The results of the two measurements  
agree within the experimental error.

Objections might be raised against the procedure  
used here to calculate the deformation potentials. In  
particular, one ought to construct the tight-binding  
functions  $d$  from resonance functions rather than from  
atomic orbitals, as discussed by Heine.<sup>39</sup> However,  
this would have little effect on the  $d$ - $sp$  overlap  $b_d$ , be-  
cause the largest contribution to this integral comes  
from regions where the resonance function and the  
atomic  $d$  function are identical (the maximum of the  
integrand lies at 0.53 of the nearest-neighbor distance).  
The calculated strain coefficients of the tight-binding  
integrals  $\sigma$ ,  $\pi$ ,  $\delta$  (Table V) are higher than predicted  
by Heine's theory, which would give  $R\partial(\ln\beta)/\partial R = -5$   
( $\beta = \sigma, \pi, \delta$ ), but their influence on the deformation po-  
tentials is small. Furthermore, it is not clear how the  
theory of Heine has to be modified if one abandons the  
muffin-tin approach, i.e., for overlapping potentials.

Two other calculations of the hydrostatic deforma-  
tion potentials are known.<sup>43,48</sup> Both are listed in

<sup>47</sup> R. Zallen, in *Colloquium on the Optical Properties and the Electronic Structure of Metals and Alloys*, Paris 1965, edited by F. Abelès (North-Holland Publishing Co., Amsterdam, 1966), p. 164.

<sup>48</sup> R. Jacobs (private communication).